Quadratics

Things you should already know

Fact (Completing the Square) —

$$x^2 + 2bx + c = (x+b)^2 + c - b^2$$

Fact (Quadratic Formula) —

$$ax^2 + bx + c = 0 \Leftrightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

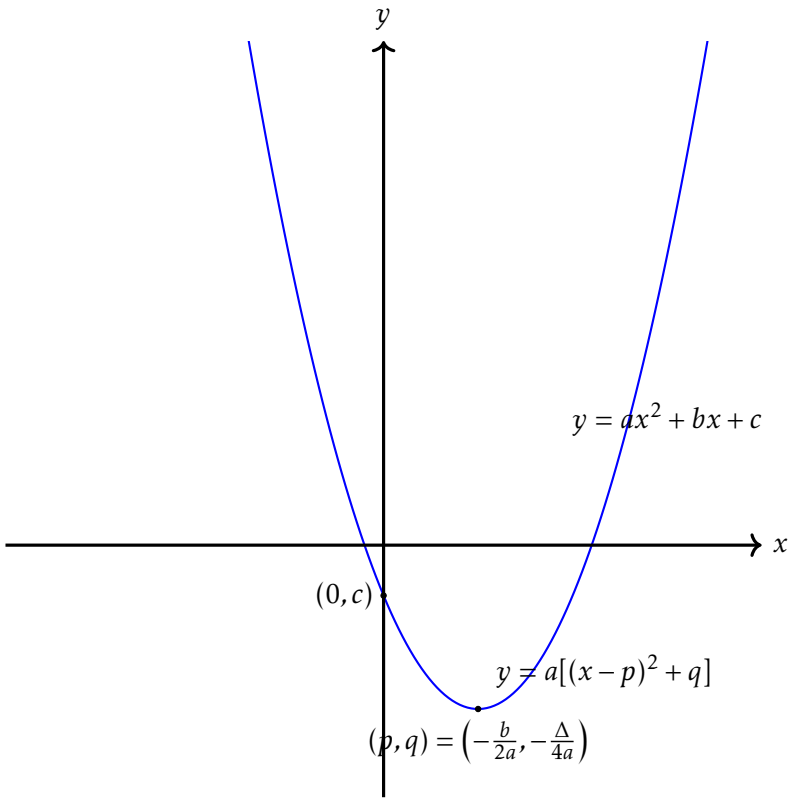
Fact (The Discriminant) —

$$\Delta = D = b^2 - 4ac$$

- $D > 0 \Rightarrow 2$ *distinct* real roots
- $D = 0 \Rightarrow 1$ *repeated* real root
- $D < 0 \Rightarrow 0$ real roots

Fact (Squares are Non-negative) —

$$x^2 \geq 0$$



Example

Prove that:

(i) if $a + 2b + 3c = 7x$, then

$$a^2 + b^2 + c^2 = (x - a)^2 + (2x - b)^2 + (3x - c)^2;$$

(ii) if $2a + 3b + 3c = 11x$, then

$$a^2 + b^2 + c^2 = (2x - a)^2 + (3x - b)^2 + (3x - c)^2.$$

Give a general result of which **(i)** and **(ii)** are special cases.

Example

Find the range of values for k for which the equation

$$x^2 - kx + (k+3) = 0 \quad \text{has real roots}$$

Example

Find the range of the function

$$g(x) = \frac{x^2 - 4x + 3}{x^2 - 6x + 10}$$

Example

Solve $x = \sqrt{x} + 12$